

Statistics of Stars in a Zone of 5° from $+65^\circ$ to $+70^\circ$ Decl. counted on Photographs for the Astrographic Chart and Catalogue at the Royal Observatory, Greenwich.

(Communicated by the Astronomer Royal.)

An important use to which the long-exposure photographs for the astrographic chart can be immediately put is that of star gauging. In combination with counts of the number of stars shown with shorter exposures, these photographs give down to their limit information of a very interesting and complete character as to the distribution of the stars.

The following paper gives the results of these counts for the zone of 5° in width between $+65^\circ$ and $+70^\circ$ Decl.

The photographs were taken with the astrographic equatorial of 33 centimetres (or 13-inch) aperture and $3^m.43$ (or 11-foot 3-inch) focus, and were usually obtained when the field photographed was within one hour of the meridian at upper culmination, and never more than one hour and a half from it. The most sensitive plates available were used—generally Ilford Special Rapid or Barnet Rocket plates. The chart plates have an exposure of 40^m , the catalogue plates three exposures of 6^m , 3^m , and 20^s respectively. Plates on any night are not considered satisfactory, and are rejected unless images of $9^m.0$ stars appear with the exposure of 20^s on the catalogue plate.

In the counting of the chart plates with 40^m exposure the same plan is followed as in the measurement of the catalogue plates with the duplex micrometer; that is, the same field of the sky (usually of one square degree) is simultaneously examined on two plates. For example, the count of stars in the part of the sky lying between 65° and 66° of declination and between $0^h 0^m$ and $0^h 9^m$ of right ascension is made at the same time for the two plates whose centres are R.A. $0^h 9^m$ Decl. 65° , and R.A. $0^h 0^m$ Decl. 66° . The number of stars seen on both plates is counted, as well as the additional stars seen (with certainty) on one plate only. The counts are all made in duplicate (by two observers independently).

The counts of the stars shown on the catalogue plates are taken from the printed, but as yet unpublished, volume of the measures for the Greenwich Astrographic Catalogue Decl. $+64^\circ$ to $+72^\circ$. In this volume the stars which are shown with an exposure of 20^s are indicated, as well as those stars which are shown with exposures of 3^m and 6^m . In the following table no discrimination has been made between the 6^m and 3^m exposures, but it is necessary to state that images of stars with 6^m exposure (which were not shown with 3^m) have not been measured unless

the star was shown on both plates. The possible combinations are :—

- (i.) 6^m and 3^m images shown on both plates.
- (ii.) 6^m and 3^m images shown on one plate, but only 6^m image on the other.
- (iii.) 6^m and 3^m images on one plate, but the star not shown on the other.
- (iv.) 6^m image, but not 3^m image shown on both plates.
- (v.) 6^m image shown on one plate but not on the other.

In the fifth of these cases the image has not been measured, and consequently these cases are not included in the counts.

The following table gives the number of stars for areas of one degree in declination and 45^m in right ascension, these numbers being usually the sum of five different areas counted on six different plates, three in one zone and three in the zone next to it. This summation tends to smooth the inequalities due to the varying conditions of the nights when the plates were taken. The area included in 45^m of right ascension in the different zones of 1° in declination is :—

| | | | | |
|------|--------|---------|---------|-----|
| 4·67 | square | degrees | in zone | 65° |
| 4·48 | „ | „ | „ | 66° |
| 4·31 | „ | „ | „ | 67° |
| 4·12 | „ | „ | „ | 68° |
| 3·94 | „ | „ | „ | 69° |

The total number of stars found in the Bonn *Durchmusterung* for these areas and of the number of stars of the ninth magnitude, or brighter, are also included in the following table.

The table gives in parallel columns the stars shown in duplicate—i.e. on each of the overlapping plates—and the total number of stars shown with the various exposures.

TABLE I.

Number of Stars shown with Various Exposures for Areas of 1° in Decl. and 45^m in R.A. in the Zones from 65° to 70° Decl.

| Limits of R.A. | Zone. | No. in B.D. | | Number shown on Photographs. | | | | | |
|--------------------------|-------|------------------------------------|--------------|---|--|---|---------------------------|---|---------------------------|
| | | 9 ^m ·0 and brighter. | Total No. | Exposure 20 ^s . Shown in du- plicate. | Exposures 3 ^m and 6 ^m . Total No. of Stars. | Exposure 40 ^m . Shown in du- plicate. | Total No. of Stars. | Exposure 40 ^m . Shown in dupli- cate. | Total No. of Stars. |
| h m 0 0 to 0 45 | + 65° | 44 | 108 | 53 | 95 | 181 | 264 | 1071 | 1390 |
| | 66 | 26 | 75 | 51 | 73 | 237 | 253 | 890 | 1138 |
| | 67 | 29 | 73 | 47 | 68 | 265 | 295 | 1336 | 1496 |
| | 68 | 17 | 60 | 50 | 78 | 282 | 308 | 1246 | 1394 |
| | + 69 | 17 | 48 | 40 | 90 | 302 | 373 | 1513 | 1680 |

K 2

| Limits of R.A. | Zone. | No. in B.D. | | Number shown on Photographs. | | | | | |
|--------------------|-------|---------------------------------|--------------|--|---|--|---------------------------|--|---------------------------|
| | | 9 ^m and brighter. | Total No. | Exposure 20 ^s . Shown in du- plicate. | Exposures 3 ^m and 6 ^m . Total No. of Stars. | Exposure 40 ^m . Shown in dupli- cate. | Total No. of Stars. | Exposure 40 ^m . Total No. of Stars. | Total No. of Stars. |
| 0 45 to 1 30 | + 65 | 35 | 100 | 69 | 140 | 258 | 347 | 1073 | 1229 |
| | 66 | 20 | 65 | 36 | 81 | 199 | 261 | 702 | 1148 |
| | 67 | 21 | 66 | 25 | 39 | 140 | 165 | 1391 | 1587 |
| | 68 | 31 | 71 | 47 | 71 | 239 | 306 | 2122 | 2324 |
| | + 69 | 23 | 61 | 55 | 122 | 332 | 413 | 2792 | 2867 |
| 1 30 to 2 15 | + 65 | 29 | 70 | 25 | 71 | 173 | 219 | 985 | 1090 |
| | 66 | 22 | 69 | 36 | 61 | 176 | 218 | 837 | 1092 |
| | 67 | 20 | 65 | 39 | 56 | 197 | 241 | 958 | 1038 |
| | 68 | 16 | 49 | 43 | 59 | 277 | 317 | 1391 | 1475 |
| | + 69 | 13 | 33 | 36 | 68 | 255 | 314 | 1799 | 1923 |
| 2 15 to 3 0 | + 65 | 20 | 64 | 21 | 53 | 136 | 176 | 976 | 1270 |
| | 66 | 12 | 37 | 13 | 19 | 62 | 68 | 720 | 842 |
| | 67 | 21 | 45 | 25 | 41 | 112 | 157 | 768 | 909 |
| | 68 | 27 | 54 | 57 | 96 | 198 | 323 | 887 | 1100 |
| | + 69 | 21 | 54 | 68 | 130 | 361 | 420 | 1262 | 1410 |
| 3 0 to 3 45 | + 65 | 23 | 46 | 36 | 75 | 147 | 193 | 527 | 633 |
| | 66 | 21 | 53 | 29 | 49 | 157 | 238 | 771 | 1041 |
| | 67 | 18 | 49 | 31 | 51 | 188 | 307 | 1107 | 1251 |
| | 68 | 18 | 78 | 45 | 85 | 250 | 305 | 1355 | 1914 |
| | + 69 | 14 | 32 | 29 | 46 | 187 | 215 | 1287 | 1719 |
| 3 45 to 4 30 | + 65 | 17 | 47 | 34 | 55 | 144 | 180 | 975 | 1222 |
| | 66 | 14 | 64 | 30 | 33 | 150 | 166 | 1149 | 1307 |
| | 67 | 19 | 45 | 29 | 52 | 158 | 249 | 1204 | 1242 |
| | 68 | 18 | 51 | 26 | 58 | 147 | 200 | 1344 | 1488 |
| | + 69 | 16 | 38 | 21 | 46 | 160 | 228 | 1127 | 1410 |
| 4 30 to 5 15 | + 65 | 16 | 47 | 47 | 67 | 230 | 303 | 2306 | 2783 |
| | 66 | 21 | 50 | 28 | 48 | 210 | 271 | 2530 | 2945 |
| | 67 | 23 | 45 | 22 | 41 | 181 | 222 | 1964 | 2461 |
| | 68 | 20 | 46 | 31 | 66 | 278 | 385 | 1620 | 1786 |
| | + 69 | 29 | 51 | 56 | 83 | 361 | 425 | 1608 | 2234 |
| 5 15 to 6 0 | + 65 | 19 | 46 | 48 | 80 | 270 | 338 | 2162 | 2871 |
| | 66 | 14 | 37 | 26 | 63 | 201 | 340 | 1513 | 2306 |
| | 67 | 11 | 30 | 26 | 46 | 181 | 260 | 1441 | 1906 |
| | 68 | 13 | 39 | 43 | 72 | 321 | 448 | 1717 | 2068 |
| | + 69 | 15 | 47 | 41 | 65 | 292 | 375 | 1748 | 2030 |

| Limits of R.A. | Zone. | No. in B.D. | | Number shown on Photographs. | | | | | |
|----------------------|-------|---------------------------------|--------------|---|--|-----------------------------|---------------------------|--|---------------------------|
| | | 9 ^m and brighter. | Total No. | Exposure 20 ^m . Shown in du- plicate. | Exposures 3 ^m and 6 ^m . Total No. of Stars. | Shown in du- plicate. | Total No. of Stars. | Exposure 40 ^m . Shown in dupli- cate. | Total No. of Stars. |
| 6 0 to 6 45 | +65 | 18 | 33 | 48 | 63 | 254 | 299 | 1364 | 2164 |
| | 66 | 16 | 44 | 48 | 77 | 324 | 372 | 1172 | 1925 |
| | 67 | 13 | 45 | 33 | 57 | 250 | 355 | 1642 | 2115 |
| | 68 | 13 | 31 | 35 | 45 | 233 | 260 | 1737 | 1960 |
| | +69 | 9 | 31 | 33 | 53 | 275 | 332 | 1655 | 1742 |
| 6 45 to 7 30 | +65 | 8 | 34 | 25 | 43 | 172 | 206 | 1465 | 1929 |
| | 66 | 21 | 43 | 37 | 60 | 201 | 272 | 1166 | 1349 |
| | 67 | 21 | 41 | 32 | 61 | 250 | 411 | 1247 | 1523 |
| | 68 | 17 | 35 | 24 | 38 | 206 | 282 | 1452 | 1537 |
| | +69 | 15 | 40 | 25 | 44 | 198 | 232 | 1235 | 1435 |
| 7 30 to 8 15 | +65 | 17 | 45 | 36 | 61 | 204 | 257 | 1155 | 1395 |
| | 66 | 17 | 32 | 30 | 58 | 219 | 258 | 1153 | 1467 |
| | 67 | 14 | 37 | 30 | 52 | 243 | 301 | 1050 | 1425 |
| | 68 | 12 | 39 | 32 | 57 | 196 | 259 | 976 | 1253 |
| | +69 | 17 | 34 | 20 | 37 | 169 | 206 | 1176 | 1370 |
| 8 15 to 9 0 | +65 | 24 | 60 | 48 | 87 | 219 | 271 | 717 | 799 |
| | 66 | 15 | 53 | 40 | 68 | 214 | 263 | 912 | 1048 |
| | 67 | 13 | 28 | 25 | 48 | 168 | 233 | 873 | 1005 |
| | 68 | 13 | 26 | 22 | 38 | 167 | 184 | 891 | 984 |
| | +69 | 10 | 43 | 38 | 59 | 206 | 271 | 989 | 1028 |
| 9 0 to 9 45 | +65 | 16 | 54 | 21 | 36 | 130 | 157 | 773 | 901 |
| | 66 | 16 | 38 | 27 | 43 | 113 | 145 | 734 | 936 |
| | 67 | 23 | 44 | 38 | 49 | 145 | 164 | 747 | 951 |
| | 68 | 5 | 26 | 23 | 35 | 155 | 169 | 736 | 790 |
| | +69 | 16 | 38 | 28 | 47 | 170 | 207 | 693 | 813 |
| 9 45 to 10 30 | +65 | 17 | 47 | 37 | 52 | 160 | 192 | 749 | 868 |
| | 66 | 13 | 39 | 26 | 39 | 166 | 180 | 850 | 986 |
| | 67 | 17 | 40 | 29 | 40 | 150 | 170 | 750 | 921 |
| | 68 | 12 | 32 | 19 | 33 | 138 | 170 | 725 | 819 |
| | +69 | 14 | 31 | 26 | 57 | 144 | 205 | 828 | 970 |
| 10 30 to 11 15 | +65 | 17 | 42 | 45 | 65 | 202 | 248 | 684 | 771 |
| | 66 | 22 | 43 | 48 | 65 | 196 | 235 | 868 | 932 |
| | 67 | 11 | 26 | 27 | 38 | 135 | 169 | 830 | 888 |
| | 68 | 9 | 28 | 31 | 37 | 152 | 172 | 802 | 836 |
| | +69 | 6 | 22 | 20 | 45 | 173 | 215 | 860 | 928 |

| Limits of R.A. | Zone. | No. in B.D. | | Number shown on Photographs. | | | | | |
|----------------------|-------|---------------------------------|--------------|--|---|---|---|--|--|
| | | 9 ^m and brighter. | Total No. | Exposure 20 ^s . Shown in du- plicate. | Exposures 20 ^s . Total No. of Stars. | Exposures 3 ^m and 6 ^m . Shown in du- plicate. | Exposures 3 ^m and 6 ^m . Total No. of Stars. | Exposure 40 ^m . Shown in dupli- cate. | Exposure 40 ^m . Total No. of Stars. |
| 11 15 to 12 0 | +65 | 13 | 38 | 25 | 41 | 131 | 151 | 757 | 885 |
| | 66 | 12 | 30 | 19 | 43 | 153 | 250 | 805 | 889 |
| | 67 | 20 | 34 | 37 | 58 | 209 | 249 | 738 | 764 |
| | 68 | 16 | 33 | 39 | 51 | 215 | 249 | 724 | 748 |
| | +69 | 19 | 37 | 47 | 89 | 249 | 346 | 812 | 945 |
| 12 0 to 12 45 | +65 | 21 | 40 | 28 | 44 | 128 | 173 | 682 | 760 |
| | 66 | 11 | 24 | 17 | 24 | 101 | 136 | 733 | 831 |
| | 67 | 9 | 35 | 22 | 38 | 137 | 187 | 686 | 797 |
| | 68 | 7 | 20 | 22 | 41 | 163 | 198 | 688 | 739 |
| | +69 | 18 | 38 | 40 | 73 | 217 | 291 | 816 | 879 |
| 12 45 to 13 30 | +65 | 17 | 38 | 33 | 57 | 156 | 191 | 801 | 953 |
| | 66 | 11 | 34 | 30 | 46 | 146 | 170 | 752 | 821 |
| | 67 | 11 | 24 | 19 | 29 | 137 | 165 | 754 | 818 |
| | 68 | 16 | 34 | 29 | 48 | 189 | 219 | 685 | 738 |
| | +69 | 10 | 24 | 34 | 62 | 244 | 303 | 732 | 766 |
| 13 30 to 14 15 | +65 | 20 | 42 | 36 | 52 | 154 | 173 | 640 | 837 |
| | 66 | 15 | 32 | 30 | 41 | 172 | 198 | 695 | 834 |
| | 67 | 20 | 46 | 33 | 42 | 164 | 210 | 775 | 888 |
| | 68 | 17 | 44 | 34 | 53 | 195 | 242 | 723 | 775 |
| | +69 | 16 | 32 | 39 | 57 | 202 | 279 | 646 | 698 |
| 14 15 to 15 0 | +65 | 25 | 52 | 38 | 59 | 141 | 208 | 831 | 902 |
| | 66 | 15 | 40 | 36 | 66 | 204 | 254 | 808 | 897 |
| | 67 | 14 | 34 | 31 | 49 | 189 | 236 | 768 | 874 |
| | 68 | 8 | 37 | 20 | 50 | 141 | 245 | 641 | 688 |
| | +69 | 18 | 41 | 28 | 75 | 134 | 283 | 672 | 758 |
| 15 0 to 15 45 | +65 | 12 | 42 | 31 | 87 | 166 | 255 | 972 | 1039 |
| | 66 | 25 | 39 | 39 | 86 | 212 | 330 | 1024 | 1133 |
| | 67 | 19 | 42 | 46 | 54 | 238 | 286 | 949 | 1142 |
| | 68 | 13 | 37 | 29 | 59 | 210 | 251 | 685 | 796 |
| | +69 | 16 | 33 | 38 | 63 | 215 | 273 | 742 | 842 |
| 15 45 to 16 30 | +65 | 26 | 49 | 49 | 76 | 175 | 245 | 1201 | 1322 |
| | 66 | 14 | 38 | 43 | 75 | 233 | 290 | 1107 | 1205 |
| | 67 | 13 | 29 | 44 | 67 | 259 | 333 | 1112 | 1216 |
| | 68 | 16 | 29 | 37 | 64 | 268 | 308 | 999 | 1035 |
| | +69 | 16 | 36 | 42 | 66 | 252 | 295 | 906 | 943 |

| Limits of R.A. | Zone. | No. in B.D. | | Number shown on Photographs. | | | | | |
|----------------------|-------|---------------------------------|--------------|--|---|---|--|---|--|
| | | 9 ^m and brighter. | Total No. | Exposure 20 ^s . Shown in du- plicate. | Exposures 3 ^m and 6 ^m . Total No. of Stars. | Exposures 3 ^m and 6 ^m . Shown in du- plicate. | Exposure 40 ^m . Total No. of Stars. | Exposures 3 ^m and 6 ^m . Shown in dupli- cate. | Exposure 40 ^m . Total No. of Stars. |
| h m | | | | | | | | | |
| 16 30 to 17 15 | +65 | 14 | 48 | 34 | 69 | 202 | 279 | 1190 | 1284 |
| | 66 | 19 | 52 | 53 | 81 | 251 | 317 | 1177 | 1287 |
| | 67 | 24 | 52 | 57 | 76 | 240 | 273 | 1368 | 1454 |
| | 68 | 18 | 43 | 37 | 65 | 259 | 301 | 1296 | 1338 |
| | +69 | 23 | 51 | 62 | 90 | 247 | 316 | 1111 | 1209 |
| 17 15 to 18 0 | +65 | 16 | 59 | 78 | 107 | 291 | 398 | 1408 | 1527 |
| | 66 | 18 | 62 | 54 | 82 | 301 | 367 | 1537 | 1935 |
| | 67 | 23 | 44 | 59 | 72 | 341 | 382 | 1839 | 1977 |
| | 68 | 24 | 48 | 64 | 93 | 372 | 397 | 1487 | 1548 |
| | +69 | 23 | 61 | 68 | 111 | 338 | 392 | 1517 | 1546 |
| 18 0 to 18 45 | +65 | 30 | 61 | 58 | 83 | 281 | 315 | 1697 | 2010 |
| | 66 | 18 | 58 | 72 | 94 | 316 | 403 | 1665 | 1744 |
| | 67 | 18 | 49 | 60 | 85 | 340 | 411 | 1359 | 1438 |
| | 68 | 18 | 51 | 46 | 87 | 310 | 446 | 1557 | 1753 |
| | +69 | 15 | 37 | 50 | 89 | 346 | 469 | 1832 | 2051 |
| 18 45 to 19 30 | +65 | 28 | 68 | 71 | 101 | 370 | 471 | 2170 | 2389 |
| | 66 | 24 | 84 | 85 | 121 | 442 | 545 | 2307 | 2441 |
| | 67 | 17 | 67 | 40 | 63 | 343 | 435 | 2249 | 2420 |
| | 68 | 17 | 51 | 57 | 102 | 406 | 479 | 2439 | 2534 |
| | +69 | 17 | 51 | 97 | 131 | 538 | 608 | 1756 | 1868 |
| 19 30 to 20 15 | +65 | 24 | 70 | 45 | 68 | 256 | 352 | 1979 | 2146 |
| | 66 | 27 | 72 | 37 | 69 | 256 | 390 | 1941 | 2022 |
| | 67 | 26 | 74 | 53 | 74 | 215 | 320 | 1300 | 1431 |
| | 68 | 15 | 49 | 41 | 58 | 290 | 326 | 1206 | 1296 |
| | +69 | 16 | 42 | 43 | 56 | 237 | 253 | 1063 | 1162 |
| 20 15 to 21 0 | +65 | 18 | 82 | 69 | 90 | 325 | 408 | 1792 | 2107 |
| | 66 | 23 | 78 | 37 | 79 | 209 | 307 | 1720 | 1816 |
| | 67 | 17 | 48 | 35 | 50 | 183 | 245 | 1003 | 1044 |
| | 68 | 17 | 57 | 47 | 78 | 322 | 378 | 1351 | 1499 |
| | +69 | 15 | 44 | 55 | 88 | 307 | 358 | 1384 | 1616 |
| 21 0 to 21 45 | +65 | 59 | 122 | 103 | 185 | 483 | 634 | 1910 | 1993 |
| | 66 | 34 | 96 | 46 | 78 | 256 | 352 | 1592 | 1749 |
| | 67 | 25 | 70 | 34 | 67 | 295 | 431 | 1535 | 1601 |
| | 68 | 21 | 74 | 57 | 97 | 391 | 441 | 1570 | 1773 |
| | +69 | 21 | 62 | 52 | 82 | 314 | 374 | 1607 | 1686 |

| Limits of R.A. | | Zone. | No. in B.D. | | Number shown on Photographs. | | | | | |
|------------------------------------|---|-------|-----------------------------------|--------------|---|--|-----------------------------|---------------------------|---|---------------------------|
| | | | 9 ^m 0 and brighter. | Total No. | Exposure 20 ^s . Shown in du- plicate. | Exposures 3 ^m and 6 ^m . Total No. of Stars. | Shown in du- plicate. | Total No. of Stars. | Exposure 40 ^m . Shown in dupli- cate. | Total No. of Stars. |
| h m | | | | | | | | | | |
| 21 45 to 22 30 | { | + 65 | 40 | 126 | 68 | 146 | 388 | 673 | 1761 | 2495 |
| | | 66 | 34 | 93 | 94 | 177 | 563 | 888 | 2249 | 2424 |
| | | 67 | 35 | 102 | 89 | 143 | 614 | 914 | 3015 | 3158 |
| | | 68 | 22 | 62 | 78 | 123 | 553 | 654 | 3504 | 3754 |
| | | + 69 | 35 | 70 | 96 | 134 | 480 | 569 | 1763 | 1908 |
| 22 30 to 23 15 | { | + 65 | 37 | 101 | 68 | 98 | 354 | 389 | 1521 | 1861 |
| | | 66 | 30 | 84 | 50 | 72 | 305 | 348 | 1605 | 1681 |
| | | 67 | 27 | 62 | 36 | 64 | 283 | 349 | 1284 | 1353 |
| | | 68 | 22 | 70 | 63 | 95 | 362 | 442 | 1312 | 1477 |
| | | + 69 | 15 | 47 | 43 | 98 | 345 | 443 | 1560 | 1870 |
| 23 15 to 0 0 | { | + 65 | 42 | 119 | 46 | 70 | 228 | 274 | 1107 | 1255 |
| | | 66 | 32 | 84 | 47 | 84 | 268 | 420 | 947 | 1162 |
| | | 67 | 32 | 86 | 48 | 79 | 286 | 461 | 1146 | 1396 |
| | | 68 | 21 | 66 | 49 | 71 | 221 | 275 | 1168 | 1408 |
| | | + 69 | 19 | 54 | 48 | 79 | 348 | 462 | 2029 | 2591 |
| Totals ... | | | 3,094 | 8,152 | 6,663 | 11,018 | 38,262 | 49,014 | 199,776 | 229,426 |
| Number per square degree. | | | 4.5 | 11.8 | 9.7 | 16.0 | 55.6 | 71.2 | 290.1 | 333.2 |
| Ratio of Totals. | | | 1 | 2.6 | 2.1 | 3.6 | 12.4 | 15.8 | 64.6 | 74.2 |

By taking the sums for the five zones the number of stars is obtained for each area of 5° in Decl., and 45^m in R.A. between Decl. +65° and +70°. Each trapezium into which this belt of the sky is thus divided contains 21.52 square degrees.

TABLE II.

Number of Stars shown with various Exposures for Areas of 5° in Declination and 45^m in Right Ascension (21.52 square degrees) between 65° and 70° N. Decl.

| Limits of R.A. | | | | Number in B.D. | | Number shown on Photographs. | | | | |
|----------------|------|--|--|--------------------------------|-----------|--|---|--|---------------------|------|
| | | | | | | Exposure 20 ^s . Shown in duplicate. | Exposures 3 ^m and 6 ^m . Total No. of Stars. | Exposure 40 ^m . Shown in duplicate. | Total No. of Stars. | |
| h m | h m | | | 9 ^m 0 and brighter. | Total No. | | | | | |
| 0 0- | 0 45 | | | 133 | 364 | 241 | 404 | 1267 | 1493 | 6056 |
| 0 45- | 1 30 | | | 130 | 363 | 232 | 453 | 1168 | 1492 | 8080 |
| 1 30- | 2 15 | | | 100 | 286 | 179 | 315 | 1078 | 1309 | 5970 |
| 2 15- | 3 0 | | | 101 | 254 | 184 | 339 | 869 | 1144 | 4613 |

| Limits of R.A. | | | | Number shown on Photographs. | | | | | | | |
|-------------------|-------|----|----|---|--------------|----------------------------|---------------------------|---|---------------------------|----------------------------|---------------------------|
| | | | | Number in B.D. | | Exposure 20 ^s . | | Exposures 3 ^m and 6 ^m . | | Exposure 40 ^m . | |
| | | | | 9 ^m 0 ^s and brighter. | Total No. | Shown in Duplicate. | Total No. of Stars. | Shown in Duplicate. | Total No. of Stars. | Shown in Duplicate. | Total No. of Stars. |
| h | m | h | m | | | | | | | | |
| 3 | 0- | 3 | 45 | 94 | 258 | 170 | 306 | 929 | 1258 | 5047 | 6558 |
| 3 | 45- | 4 | 30 | 84 | 245 | 140 | 244 | 759 | 1023 | 5799 | 6669 |
| 4 | 30- | 5 | 15 | 109 | 239 | 184 | 305 | 1260 | 1606 | 10,028 | 12,209 |
| 5 | 15- | 6 | 0 | 72 | 199 | 184 | 326 | 1265 | 1761 | 8581 | 11,181 |
| 6 | 0- | 6 | 45 | 69 | 184 | 197 | 295 | 1336 | 1618 | 7570 | 9906 |
| 6 | 45- | 7 | 30 | 82 | 193 | 143 | 246 | 1027 | 1403 | 6565 | 7773 |
| 7 | 30- | 8 | 15 | 77 | 187 | 148 | 265 | 1031 | 1281 | 5510 | 6910 |
| 8 | 15- | 9 | 0 | 75 | 210 | 173 | 300 | 974 | 1222 | 4382 | 4864 |
| 9 | 0- | 9 | 45 | 76 | 200 | 137 | 210 | 713 | 842 | 3683 | 4391 |
| 9 | 45-10 | 30 | | 73 | 189 | 137 | 221 | 758 | 917 | 3902 | 4564 |
| 10 | 30-11 | 15 | | 65 | 161 | 171 | 250 | 858 | 1039 | 4044 | 4355 |
| 11 | 15-12 | 0 | | 80 | 172 | 167 | 282 | 957 | 1245 | 3836 | 4231 |
| 12 | 0-12 | 45 | | 66 | 157 | 129 | 220 | 746 | 985 | 3605 | 4006 |
| 12 | 45-13 | 30 | | 65 | 154 | 145 | 242 | 872 | 1048 | 3724 | 4096 |
| 13 | 30-14 | 15 | | 88 | 196 | 172 | 245 | 887 | 1102 | 3479 | 4032 |
| 14 | 15-15 | 0 | | 80 | 204 | 153 | 299 | 809 | 1226 | 3720 | 4119 |
| 15 | 0-15 | 45 | | 85 | 193 | 183 | 349 | 1041 | 1395 | 4372 | 4952 |
| 15 | 45-16 | 30 | | 85 | 181 | 215 | 348 | 1187 | 1471 | 5325 | 5721 |
| 16 | 30-17 | 15 | | 98 | 246 | 243 | 381 | 1199 | 1486 | 6142 | 6572 |
| 17 | 15-18 | 0 | | 104 | 274 | 323 | 465 | 1643 | 1936 | 7788 | 8533 |
| 18 | 0-18 | 45 | | 99 | 256 | 286 | 438 | 1593 | 2044 | 8110 | 8996 |
| 18 | 45-19 | 30 | | 103 | 321 | 350 | 518 | 2099 | 2538 | 10,921 | 11,652 |
| 19 | 30-20 | 15 | | 108 | 307 | 219 | 325 | 1254 | 1641 | 7489 | 8057 |
| 20 | 15-21 | 0 | | 90 | 309 | 243 | 385 | 1346 | 1696 | 7250 | 8082 |
| 21 | 0-21 | 45 | | 160 | 424 | 292 | 509 | 1739 | 2232 | 8214 | 8802 |
| 21 | 45-22 | 30 | | 166 | 453 | 425 | 723 | 2598 | 3698 | 12,292 | 13,739 |
| 22 | 30-23 | 15 | | 131 | 364 | 260 | 427 | 1649 | 1971 | 7282 | 8242 |
| 23 | 15- 0 | 0 | | 146 | 409 | 238 | 383 | 1351 | 1892 | 6397 | 7812 |
| Total ... | | | | 3094 | 8152 | 6663 | 11,018 | 38,262 | 49,014 | 199,776 | 229,426 |

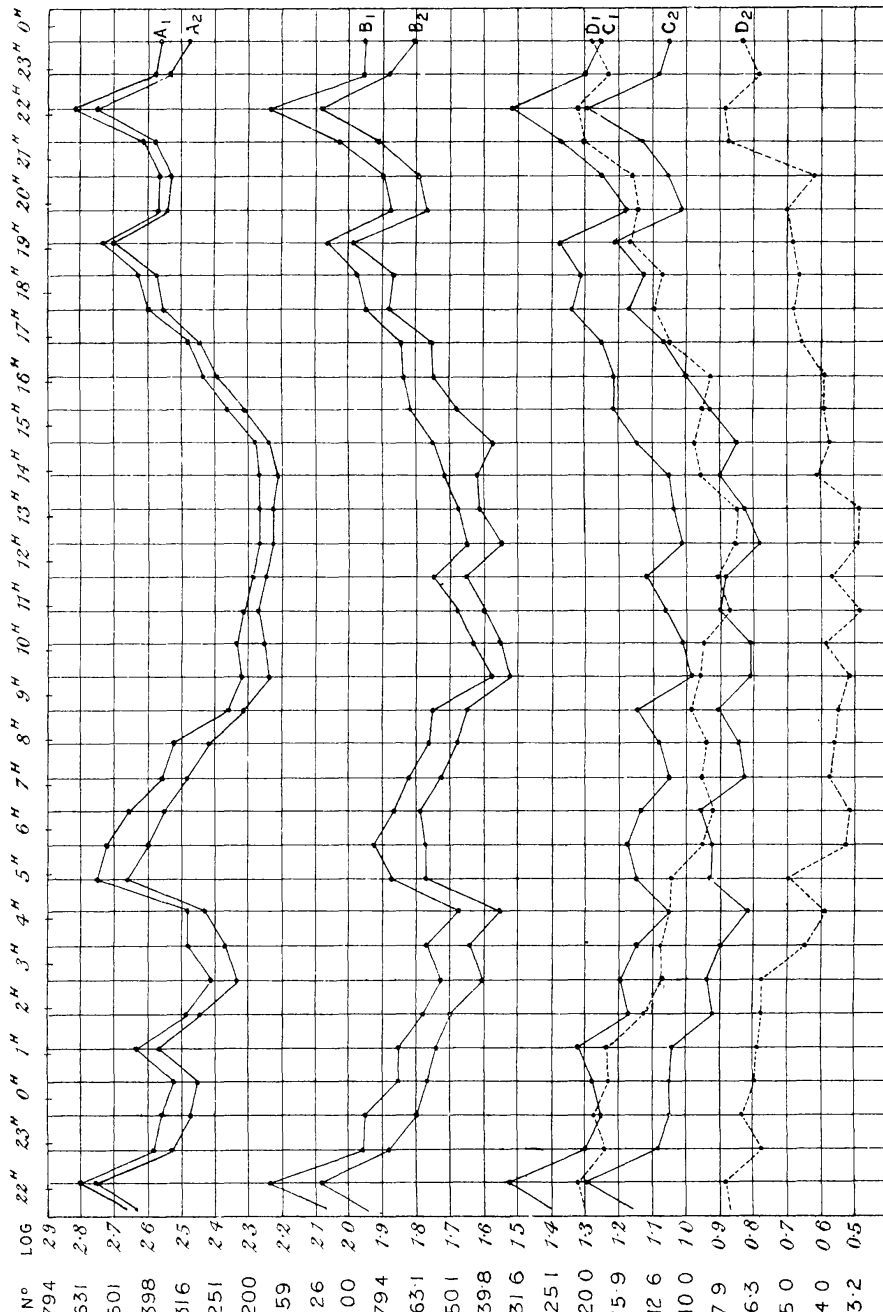
In Table III. the logarithms of these numbers are given, and in the diagram the logarithm of the number of stars per square degree is given, the numbers of the diagram being obtained from Table III. by subtracting logarithm 21.5 or 1.332.

TABLE III.

Logarithm of Number of Stars shown with Various Exposures for Areas of 5° in Decl. and 45^m in R.A. between the Limits + 65° and + 70° of Decl.

| Limits of R.A. | | | | Number shown on Photographs. | | | | | | | |
|----------------|-----|----|----|---|-----------|----------------------------|-----------|---|-----------|----------------------------|-----------|
| | | | | Number in B.D. | | Exposure 20 ^s . | | Exposures 3 ^m and 6 ^m . | | Exposure 40 ^m . | |
| | | | | 9 ^m 0 ^s and brighter. | Total No. | Shown in Duplicate. | Total No. | Shown in Duplicate. | Total No. | Shown in Duplicate. | Total No. |
| h | m | h | m | | | | | | | | |
| 0 | 0- | 0 | 45 | 2.124 | 2.561 | 2.382 | 2.606 | 3.103 | 3.174 | 3.782 | 3.851 |
| 0 | 45- | 1 | 30 | 2.114 | 2.560 | 2.366 | 2.656 | 3.067 | 3.174 | 3.907 | 3.962 |
| 1 | 30- | 2 | 15 | 2.000 | 2.456 | 2.253 | 2.498 | 3.033 | 3.117 | 3.776 | 3.821 |
| 2 | 15- | 3 | 0 | 2.004 | 2.405 | 2.265 | 2.530 | 2.939 | 3.058 | 3.664 | 3.743 |
| 3 | 0- | 3 | 45 | 1.973 | 2.412 | 2.230 | 2.486 | 2.968 | 3.100 | 3.703 | 3.817 |
| 3 | 45- | 4 | 30 | 1.924 | 2.389 | 2.146 | 2.387 | 2.880 | 3.010 | 3.763 | 3.824 |
| 4 | 30- | 5 | 15 | 2.037 | 2.378 | 2.265 | 2.484 | 3.100 | 3.206 | 4.001 | 4.087 |
| 5 | 15- | 6 | 0 | 1.857 | 2.299 | 2.265 | 2.513 | 3.102 | 3.246 | 3.934 | 4.048 |
| 6 | 0- | 6 | 45 | 1.839 | 2.265 | 2.294 | 2.470 | 3.126 | 3.209 | 3.879 | 3.996 |
| 6 | 45- | 7 | 30 | 1.914 | 2.286 | 2.155 | 2.391 | 3.012 | 3.147 | 3.817 | 3.891 |
| 7 | 30- | 8 | 15 | 1.887 | 2.272 | 2.170 | 2.423 | 3.013 | 3.107 | 3.741 | 3.839 |
| 8 | 15- | 9 | 0 | 1.875 | 2.322 | 2.238 | 2.477 | 2.989 | 3.087 | 3.642 | 3.687 |
| 9 | 0- | 9 | 45 | 1.881 | 2.301 | 2.137 | 2.322 | 2.853 | 2.925 | 3.566 | 3.643 |
| 9 | 45- | 10 | 30 | 1.863 | 2.276 | 2.137 | 2.344 | 2.880 | 2.962 | 3.591 | 3.659 |
| 10 | 30- | 11 | 15 | 1.813 | 2.207 | 2.233 | 2.398 | 2.933 | 3.017 | 3.607 | 3.639 |
| 11 | 15- | 12 | 0 | 1.903 | 2.236 | 2.223 | 2.450 | 2.981 | 3.095 | 3.584 | 3.626 |
| 12 | 0- | 12 | 45 | 1.819 | 2.196 | 2.111 | 2.342 | 2.873 | 2.993 | 3.557 | 3.603 |
| 12 | 45- | 13 | 30 | 1.813 | 2.188 | 2.161 | 2.384 | 2.940 | 3.020 | 3.571 | 3.612 |
| 13 | 30- | 14 | 15 | 1.945 | 2.292 | 2.235 | 2.389 | 2.948 | 3.042 | 3.541 | 3.605 |
| 14 | 15- | 15 | 0 | 1.903 | 2.310 | 2.185 | 2.476 | 2.908 | 3.088 | 3.570 | 3.615 |
| 15 | 0- | 15 | 45 | 1.929 | 2.286 | 2.262 | 2.543 | 3.017 | 3.145 | 3.641 | 3.695 |
| 15 | 45- | 16 | 30 | 1.929 | 2.258 | 2.332 | 2.542 | 3.074 | 3.168 | 3.726 | 3.757 |
| 16 | 30- | 17 | 15 | 1.991 | 2.391 | 2.386 | 2.581 | 3.079 | 3.172 | 3.788 | 3.818 |
| 17 | 15- | 18 | 0 | 2.017 | 2.438 | 2.509 | 2.667 | 3.216 | 3.287 | 3.891 | 3.931 |
| 18 | 0- | 18 | 45 | 1.996 | 2.408 | 2.456 | 2.641 | 3.202 | 3.310 | 3.909 | 3.954 |
| 18 | 45- | 19 | 30 | 2.013 | 2.506 | 2.544 | 2.714 | 3.322 | 3.404 | 4.038 | 4.066 |
| 19 | 30- | 20 | 15 | 2.033 | 2.487 | 2.340 | 2.512 | 3.098 | 3.215 | 3.874 | 3.906 |
| 20 | 15- | 21 | 0 | 1.954 | 2.490 | 2.386 | 2.585 | 3.129 | 3.229 | 3.860 | 3.907 |
| 21 | 0- | 21 | 45 | 2.204 | 2.627 | 2.465 | 2.707 | 3.240 | 3.349 | 3.914 | 3.945 |
| 21 | 45- | 22 | 30 | 2.220 | 2.656 | 2.628 | 2.859 | 3.415 | 3.568 | 4.090 | 4.138 |
| 22 | 30- | 23 | 15 | 2.117 | 2.561 | 2.415 | 2.630 | 3.217 | 3.295 | 3.862 | 3.916 |
| 23 | 15- | 0 | 0 | 2.164 | 2.612 | 2.377 | 2.583 | 3.131 | 3.277 | 3.806 | 3.893 |
| Mean | ... | | | 1.970 | 2.385 | 2.298 | 2.518 | 3.056 | 3.162 | 3.769 | 3.828 |

Diagram giving number of Stars per square degree shown on Plates with various exposures, between 65° and 70° North Declination.



A₁ Total number of Stars shown with an exposure of 40^m.
B₁ Total number of Stars shown with exposures of 3^m & 6^m.
C₁ Total number of Stars shown with an exposure of 20^s.
D₁ Total number shown in Bonn Durchmusterung.

A₂ Number shown in duplicate, i.e., on both of the overlapping places.
B₂ Number shown in duplicate, i.e., on both of the overlapping places.
C₂ Number shown in duplicate, i.e., on both of the overlapping places.
D₂ Number in B.D. of magnitude 9^o and brighter.

Examination of the tables and diagram (Plate 7) shows the following results:—

(i.) The logarithm of the ratio of the greatest to the least number of stars per square degree is

0·54 or 0·55 for the 40^m exposure
 0·56 or 0·64 for the 3^m and 6^m exposures
 0·51 or 0·52 for the 20^s exposures
 and 0·47 for the Bonn *Durchmusterung*.

where the first figure in each case is derived from the number of stars counted in duplicate and the second from the total number of stars. These figures show that the maximum number of stars per square degree rises to 3·5 times the minimum number for the 40^m exposure, about 4·0 times for the 3^m and 6^m exposures, and to about 3·2 times for the 20^s exposures.

(ii.) The numbers of the stars for the exposures 40^m, 6^m and 3^m, and 20^s maintain the same ratio for different parts of the sky. The only notable exception is the increase in the numbers of the 6^m and 3^m exposures, and more still of the 40^m exposures from 5^h to 6^h. This is not shown by the 20^s exposures.

(iii.) The variation in the number of stars per square degree given in the Bonn *Durchmusterung* is on the whole well supported by the short-exposure photographs. There is not in this region any marked falling off in the B.D. where the stars are rich, though there appears to be a slight deficiency at the maximum near 22^h. Relatively to the photographs the B.D. is rich in stars from 23^h to 5^h, and poor from 12^h to 19^h.

(iv.) Generally the number of stars shown with 20^s exposure is considerably in excess of the total number given in the Bonn *Durchmusterung*, while the number shown in duplicate (on both of the two overlapping plates) is generally less.

(v.) The ratio of the total number of stars shown to the number which are shown in duplicate on both overlapping plates is very constant for each of the three different series of exposures, but apparently differs from one to another. These ratios are :

| Exposure. | Log. Ratio. | Ratio. |
|-----------------------------------|-------------|--------|
| 20 ^s | ·220 | 1·66 |
| 3 ^m and 6 ^m | ·106 | 1·28 |
| 40 ^m | ·059 | 1·15 |

The cause of the smaller ratio for the 3^m and 6^m exposures is, as will be shown in the next paragraph, that the figures and diagrams given for the 6^m and 3^m exposures in the case of the "total number" of stars nearly correspond to the 3^m images, but do not for the number "shown in duplicate."

For the 40^m exposure the smallness of the ratio is probably due to the caution of the observers, who were instructed not to include any doubtful images seen only on one plate unless they were absolutely convinced that these were not photographic defects.

As it seemed of interest to obtain a formula giving the number of stars per square degree in terms of the duration of the exposure, a further analysis has been made of the number of stars shown with 3^m exposure for Zone 69°. These numbers are given in the following table compared with the corresponding numbers of Table I. The logarithms of their ratios are also given.

Number of Stars in Zone 69° shown with an Exposure of 3^m compared with the Number previously tabulated of those shown with 3^m and 6^m Exposures.

| Limits of R.A. h m h m | | | | Total Number. | | Log. Ratio. | Number. | | Log. Ratio. |
|---------------------------|-----|----|----|------------------------------------|------------------|----------------|------------------------------------|------------------|----------------|
| | | | | Exposure | | | Exposure | | |
| | | | | 3 ^m or 6 ^m . | 3 ^m . | | 3 ^m or 6 ^m . | 3 ^m . | |
| 0 | 0- | 0 | 45 | 373 | 347 | ·031 | 302 | 199 | ·181 |
| 0 | 45- | 1 | 30 | 413 | 399 | ·015 | 332 | 226 | ·167 |
| 1 | 30- | 2 | 15 | 314 | 304 | ·014 | 255 | 186 | ·137 |
| 2 | 15- | 3 | 0 | 420 | 408 | ·012 | 361 | 277 | ·116 |
| 3 | 0- | 3 | 45 | 215 | 208 | ·014 | 187 | 152 | ·090 |
| 3 | 45- | 4 | 30 | 228 | 217 | ·022 | 160 | 96 | ·222 |
| 4 | 30- | 5 | 15 | 425 | 391 | ·036 | 361 | 250 | ·160 |
| 5 | 15- | 6 | 0 | 375 | 360 | ·018 | 292 | 220 | ·123 |
| 6 | 0- | 6 | 45 | 332 | 323 | ·012 | 275 | 199 | ·140 |
| 6 | 45- | 7 | 30 | 232 | 227 | ·009 | 198 | 148 | ·127 |
| 7 | 30- | 8 | 15 | 206 | 204 | ·004 | 169 | 137 | ·091 |
| 8 | 15- | 9 | 0 | 271 | 268 | ·005 | 206 | 163 | ·102 |
| 9 | 0- | 9 | 45 | 207 | 200 | ·015 | 170 | 118 | ·158 |
| 9 | 45- | 10 | 30 | 205 | 201 | ·009 | 144 | 96 | ·176 |
| 10 | 30- | 11 | 15 | 215 | 212 | ·006 | 173 | 122 | ·152 |
| 11 | 15- | 12 | 0 | 346 | 341 | ·006 | 249 | 207 | ·080 |
| 12 | 0- | 12 | 45 | 291 | 283 | ·012 | 217 | 174 | ·095 |
| 12 | 45- | 13 | 30 | 303 | 297 | ·009 | 244 | 190 | ·108 |
| 13 | 30- | 14 | 15 | 279 | 275 | ·006 | 202 | 138 | ·165 |
| 14 | 15- | 15 | 0 | 283 | 282 | ·002 | 134 | 105 | ·106 |
| 15 | 0- | 15 | 45 | 273 | 268 | ·008 | 215 | 158 | ·133 |
| 15 | 45- | 16 | 30 | 295 | 290 | ·008 | 252 | 209 | ·081 |
| 16 | 30- | 17 | 15 | 316 | 312 | ·006 | 247 | 209 | ·073 |
| 17 | 15- | 18 | 0 | 392 | 379 | ·014 | 338 | 266 | ·104 |
| 18 | 0- | 18 | 45 | 469 | 463 | ·005 | 346 | 281 | ·090 |
| 18 | 45- | 19 | 30 | 608 | 592 | ·012 | 538 | 419 | ·109 |
| 19 | 30- | 20 | 15 | 253 | 248 | ·009 | 237 | 172 | ·139 |
| 20 | 15- | 21 | 0 | 358 | 346 | ·015 | 307 | 231 | ·123 |
| 21 | 0- | 21 | 45 | 374 | 356 | ·022 | 314 | 221 | ·153 |
| 21 | 45- | 22 | 30 | 569 | 551 | ·014 | 480 | 392 | ·088 |
| 22 | 30- | 23 | 15 | 443 | 438 | ·005 | 345 | 261 | ·121 |
| 23 | 15- | 0 | 0 | 462 | 456 | ·006 | 348 | 264 | ·120 |
| Mean | | | | | | ·012 | ·126 | | |

The above table shows that the total number of stars which give images with 3^m exposure on one of the two plates for each area is only very slightly less than the total number given in Tables I., II., III., and in the diagram as corresponding to an exposure of 3^m or 6^m, the difference .012 of the logs. corresponding to a ratio of 0.97 to 1. This is due to the method adopted in the measurement of rejecting single images seen only on one plate (see p. 121). There is, however, a very considerable difference in the number of stars shown on both plates with an exposure of 3^m and the corresponding numbers tabulated above for 3^m and 6^m; the difference of .126 in the logarithms corresponding to a ratio of .75 to 1.00.

Assuming that these figures, which are actually found only for Zone 69°, apply to the five zones, we have for the logarithms of the number of stars per square degree with the different exposures—

I. *Shown on both Plates.*

| 20 ^s . | 3 ^m . | 6 ^m . | 40 ^m . |
|-------------------|------------------|------------------|-------------------|
| 0.966 | 1.598 | 1.724 | 2.437 |

II. *Total Number shown.*

| 20 ^s . | 3 ^m . | 6 ^m . | 40 ^m . |
|-------------------|------------------|------------------|-------------------|
| 1.186 | 1.818 | 1.830 | 2.496 |

It would seem that these numbers are accurate in the first case, but that in the second case not all the stars shown with 6^m exposure are counted, and probably not all of those shown with an exposure of 40^m. The difference of the logarithms of the number of stars shown with an exposure of 3^m and with an exposure of 20^s is .632 both for the number of stars shown in duplicate and for the total number, while the difference for the exposures 40^m and 20^s is 1.471 for the stars shown in duplicate and 1.310 for the total number. Assuming the formula

$\log \frac{N}{N_0} = k \log \frac{T}{T_0}$, where N is the number of stars per square degree shown with duration of exposure T , the interval 20^s to 3^m gives $k = .66$, and the interval 20^s to 40^m gives $k = .70$ for stars shown in duplicate and .63 for the total number. Thus we may take $\log \frac{N}{N_0} = .67 \log \frac{T}{T_0}$ or $\left(\frac{N}{N_0}\right)^2 = \left(\frac{T}{T_0}\right)^3$ as a good approximate formula for the increase of numbers with exposure between the limits of 20^s and 40^m.

On the assumption that an equal total amount of light produces an equal photographic effect, an additional magnitude is reached by increasing the exposure 2.5 times. Between the 3^m and 20^s (ratio of 9 to 1) there corresponds a difference of magnitude 2^m.36. Between the 40^m and 20^s exposure (ratio 120 to 1) there is a difference of 5^m.20. If r be the ratio of the

number of stars down to magnitude $m + 1$ to the number down to magnitude m , we obtain

$$r^{2.38} = 4.29 [\log = .632] \text{ from the } 20^s \text{ and } 3^m \text{ images} \}$$

$$r^{5.20} = 29.58 [\log = 1.471] \text{ from the } 20^s \text{ and } 40^m \text{ images} \}$$

$$\text{Thus from the } 20^s \text{ and } 3^m \text{ images we get } r = 1.84 \}$$

$$\text{and from the } 20^s \text{ and } 40^m \text{ images we get } r = 1.92 \}$$

These ratios are deduced from the number of stars shown in duplicate.

The variation of the star density with the distance from the Milky Way is distinctly shown, though not in very striking manner by these counts, as the area considered only just reaches the Milky Way at about 0^h , and is 50° from it at 12^h . The minimum number of stars per square degree, especially for the 40^m exposure, is very noticeable from 9^h to 15^h . Here the distance from the Milky Way lies between 50° to 43° . There is a gradual rise to about three times this number of stars at 5^h and 19^h , when the centre of the zone considered approaches closely to the Milky Way, and there is a still more pronounced maximum at 22^h , which is strikingly shown with all three exposures. Between 19^h and 5^h , where the zone approaches and just enters the galaxy, there are, as the diagram shows, some irregularities with two maxima and three minima.

Note on the Reproduction and Publication of the Photographs for the Astrographic Chart taken at the Royal Observatory, Greenwich.

(Communicated by the Astronomer Royal.)

In accordance with the recommendation of the International Committee, it is proposed to reproduce and distribute to a limited number of Observatories and Institutions the Greenwich portion of the Astrographic Chart, the original negatives being enlarged to twice the scale.

After careful trial a method of direct photographic reproduction has been adopted, prints being taken by contact on bromide card (15 in. \times 12 in.) from an enlarged positive on glass. By this method it is found that practically no stars are lost in reproduction, and that there is little liability to the introduction of false stars in the process beyond those which may exist in the original negative, and these can readily be detected by reference to the overlapping plate. They are for the